## Exercise 58

The accompanying figure shows the graph of a function $g(t)$ with domain $[-4,0]$ and range $[-3,0]$. Find the domains and ranges of the following functions, and sketch their graphs.

a. $g(-t)$
b. $-g(t)$
c. $g(t)+3$
d. $1-g(t)$
e. $g(-t+2)$
f. $g(t-2)$
g. $g(1-t)$
h. $-g(t-4)$

## Solution

Part a.

$$
g(-t)
$$

Replacing $t$ with $-t$ reflects the graph of $g(t)$ about the $y$-axis.


The domain of $g(-t)$ is $[0,4]$, and the range of $g(-t)$ is $[-3,0]$.

## Part b.

$$
-g(t)
$$

Multiplying $g(t)$ by -1 reflects the graph over the $t$-axis.


The domain of $-g(t)$ is $[-4,0]$, and the range of $-g(t)$ is $[0,3]$.

## Part c.

$$
g(t)+3
$$

Adding 3 to $g(t)$ shifts the graph up by 3 units.


The domain of $g(t)+3$ is $[-4,0]$, and the range of $g(t)+3$ is $[0,3]$.

## Part d.

$$
1-g(t)
$$

Multiplying $g(t)$ by -1 reflects the graph over the $t$-axis, and adding 1 shifts the graph up by 1 unit.


The domain of $1-g(t)$ is $[-4,0]$, and the range of $1-g(t)$ is $[1,4]$.

## Part e.

$$
\begin{gathered}
g(-t+2) \\
g(-(t-2))
\end{gathered}
$$

Replacing $t$ with $-t$ reflects the graph of $g(t)$ about the $y$-axis, and replacing $t$ with $t-2$ shifts the graph to the right by 2 units.


The domain of $g(-t+2)$ is $[2,6]$, and the range of $g(-t+2)$ is $[-3,0]$.

## Part f.

$$
g(t-2)
$$

Replacing $t$ with $t-2$ shifts the graph to the right by 2 units.


The domain of $g(t-2)$ is $[-2,2]$, and the range of $g(t-2)$ is $[-3,0]$.
Part g.

$$
\begin{gathered}
g(1-t) \\
g(-(t-1))
\end{gathered}
$$

Replacing $t$ with $-t$ reflects the graph of $g(t)$ about the $y$-axis, and replacing $t$ with $t-1$ shifts the graph to the right by 1 unit.


The domain of $g(1-t)$ is $[1,5]$, and the range of $g(1-t)$ is $[-3,0]$.

## Part h.

$$
-g(t-4)
$$

Multiplying $g(t)$ by -1 reflects the graph over the $t$-axis, and replacing $t$ with $t-4$ shifts the graph to the right by 4 units.


The domain of $-g(t-4)$ is $[0,4]$, and the range of $-g(t-4)$ is $[0,3]$.

